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THE STABILITY OF POWERED FLIGHT AROUND ASTEROIDS WITH APPLICATION TO VESTA

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The reliability of low-thrust trajectories between science orbits around large asteroids must be evaluated subject to the unavoidable uncertainties of orbit determination, asteroid physical parameters, momentum de-saturation maneuvers, and transfer maneuver execution error. This paper presents a computationally inexpensive way to extend the concept or orbital stability to trajectories undergoing continuously powered low-thrust flight. Trajectories that are stable using this measure are shown to be stable under the combined uncertainties expected during operations. The measure is general and relatively simple to implement. The method was applied to maneuvers planed around the asteroid Vesta in support of NASA's Dawn Discovery mission.

INTRODUCTION

The science objectives of the NASA Dawn Discovery mission are to explore the largest two members of the main asteroid belt.[1] Both Vesta and Ceres are proto planets that failed to grow to a much larger size as a result of the gravitational disruption caused by the rapid formation of Jupiter. The NASA Dawn Discovery mission to Vesta and Ceres will arrive in orbit at Vesta in August of 2011. Dawn uses solar electric propulsion (low-thrust) for capture and all orbital transfers. Four different near polar science orbits are planned at Vesta. The science orbits are Survey orbit at a radius of near 3000 [km], High Altitude Mapping Orbit (HAMO) at a radius near 950 [km], Low Altitude Mapping Orbit at a radius of near 460 [km], and High Altitude Mapping Orbit 2 (HAMO2) again at a radius near 950 [km] but at a later epoch than HAMO. The transfers between science orbits require long periods of near continuous thrusting resulting in multi-revolution spiral trajectories. Vesta is the largest main belt asteroid in the solar system. Its irregular shape and large size are expected to result in a very complex and relatively strong gravity field. The irregular, strong gravity coupled with the low control authority of solar electric propulsion results in a unique challenge for Dawn maneuver design.

The reliability of candidate trajectories between science orbits must be evaluated against the unavoidable uncertainties of orbit determination, central body physical parameters, momentum de-saturation maneuvers, and transfer maneuver execution. This paper presents a relatively inexpensive way to extend the concept of orbital stability to trajectories undergoing nearly continuously powered flight. Trajectories that are stable using the measure presented in this paper have been shown to be stable under the combined nonlinear uncertainties expected to be present during Vesta operations. The measure is generally applicable and relatively simple to implement.

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In this paper, instability is defined as a rapidly increasing state difference between a powered reference trajectory and the operationally achieved trajectory following an open loop thrust control sequence. In general, an unstable powered trajectory is one that rapidly diverges from its designed reference when subject to perturbations in position and/or velocity.

For many applications, stability analysis cannot ignore nonlinear dynamics. The application to Vesta is a particular example. The dynamical regime is highly nonlinear during the transfers between the two lowest orbits (HAMO and LAMO.) Orbital period - rotational period resonances result in a strong coupling between the Vesta rotational energy and angular momentum and the Dawn spacecraft's orbital energy and angular momentum. The transfer from HAMO to LAMO and then up again requires passage through the very strong 1:1 resonance. Finding a stable powered trajectory through this resonance is critical for mission reliability and safety.

It is necessary to operate the Dawn spacecraft with open loop thrust control laws for periods of time. The maximum duration an open loop control is useful is limited by, among other things, the inherent powered flight stability of the reference trajectory. An example of a powered spiral transfer from HAMO to LAMO is provided in Figure 1. The different colors indicate different open loop thrust (design) periods. In this example there are 9 open loop control periods covering a transfer duration of 36.8 days. The transfer in Figure 1 requires a total of 153.3 revolutions around Vesta. The longest possible duration and timing of the design periods is very difficult to determine. Long periods are desired to reduce operational complexity. But the length of each design period is limited by the powered flight stability and the expected orbit determination and maneuver execution uncertainties. The maximum size of the final open loop design cycle is further constrained by the required accuracy of the delivery to the destination science orbit.

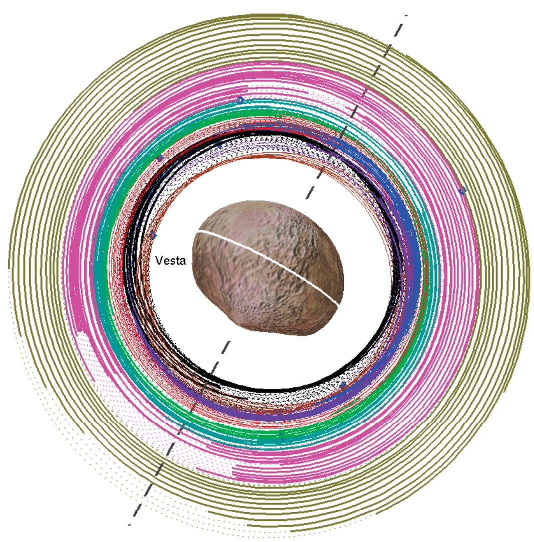

Figure 1. A candidate spiral transfer around Vesta from the High Altitude Mapping Orbit (HAMO) to the Low Altitude Mapping Orbit (LAMO). Different colors indicate different design (open-loop control) cycles. Solid lines indicate powered flight and dotted lines indicate coasting periods.

The importance of nonlinear effects precludes the use of methods which linearize the dynamics about the reference trajectory. From a practical point of view, it is necessary to investigate many alternate partitionings of transfers into open loop control periods to find designs which will be robust for operations. For the Dawn mission, some investigations have been done using fully non-linear trajectory propagation and non-linear optimization in a Monte Carlo analysis.[2] The nonlinear optimization method used in the Monte Carlo analysis and also used for all of Dawn's trajectory design is the Static/Dynamic Optimal Control method[3] embodied in the Mystic[4] software set. The fully non-linear Monte Carlo optimization approach is very computationally expensive. It was this great computational expense that lead to a search for a rapid screening method to reduce the number of fully nonlinear Monte Carlo optimization experiments that needed to be run on candidate trajectories. The screening method developed here is applied to the entire transfer (like the one in Figure 1). Periods of low stability are easily identified. Short open loop thrust control periods can be applied during low stability periods or a new more stable reference can be searched for.

APPROACH

The first step is to estimate a "typical" distribution of expected trajectory perturbations in position and velocity (phase) space. To obtain this distribution, a Monte Carlo simulation of nonlinear propagations, without the expense of including non-linear optimization, can be used. The propagations should include sampled orbit determination error, gravity knowledge error, momentum de-saturation uncertainty, and transfer maneuver execution error. The resulting spread of trajectory end-points in phase space can be fit with a mean vector and a covariance. The propagation duration should be similar to the expected time between the orbit determination data cut off for a typical open loop maneuver design and the time when the open loop maneuver design begins to execute. For the Dawn mission at Vesta, it is typically 3 days. The method described here is not very sensitive to the assumed distribution of expected trajectory perturbations. The main purpose of the distribution is to provide an approximate shape of typical dispersions in phase space. For example, down-track perturbations are typically much larger than cross track perturbations for orbital missions. A covariance matrix model developed in the way described above will exhibit this shape.

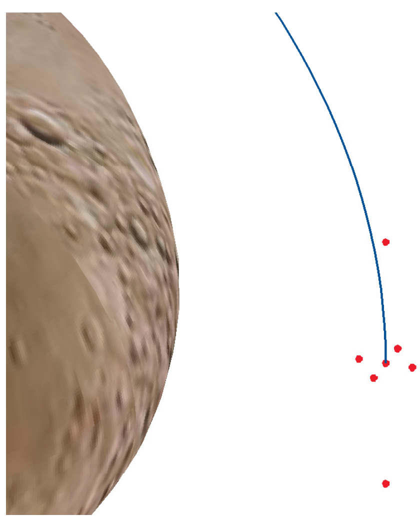

Figure 2. An example of 6 position displacements developed from eigen vectors. The position displacements are centered on a blue reference trajectory. The size of the displacements are exaggerated for illustrative purposes.

The second step is to compute six independent Eigen vectors of the covariance and also record the reverse directions of these six Eigen vectors. The result is twelve displacements in the six dimensional position-velocity phase space corresponding to a single standard deviation perturbation. Using these twelve displacements rather than randomly sampling the covariance greatly reduces the required number of displacements need to characterize the phase space. This approach is similar to the previously developed sigma point methods for Kalman filters[5] and unscented filtering methods for nonlinear estimation.[6] Figure 2 is an example of 6 position space displacements about the reference propagation (blue trajectory). The most useful frame in which to compute the covariance and resulting eigen vectors is in the Radial, Transverse, and Normal (RTN) frame. The RTN frame is defined as follows. The radial direction is defined as the (outward) directed direction from the central body to the spacecraft, the transverse direction is defined as the local horizontal direction closest to the orbital velocity, and the normal direction completes the right handed, orthonormal coordinate system. The advantage of using this frame is that the resulting twelve displacements in phase space can be applied at any point around an orbit.

The third and final step is to use a single set of twelve displacements in phase space to create multiple state clouds around a candidate reference trajectory at many different discrete times. A single set of displacements in the RTN frame can be re-used many times as long as the reference trajectory does not change dramatically in orbital period (changing by more than a factor of 2). The state clouds are then non-linearly propagated forward using the reference trajectory thrust sequence (open loop). The divergence of each of the state clouds from the original reference trajectory as a function of time during the powered and coasting portions of the reference trajectory is recorded. Relatively rapid divergence of one or more cloud states indicates the powered flight is unstable during this time period.

A criteria for "stable enough" will depend on the required minimum size of the open loop operation periods. For example, if the goal is to have 6 day or longer open loop periods, then very little phase error should occur during any 6 day period. Generally, if the reference orbit phase and the perturbed open loop thrusting phasing get far enough apart, either a shorter open loop control period must be used or a different (more stable) reference trajectory must be found. In practice, highly unstable parts of low-thrust transfers are quite obvious. The computational effort of this process is limited to 12 propagations per sample time. Sample time spacing of 12 or 24 hours has been found to be sufficient to adequately cover the 40-day, 170 revolution, spiral transfer from HAMO to LAMO.

APPLICATION TO DAWN'S MISSION AT VESTA

The HAMO to LAMO and LAMO to HAMO2 transfers represent the most important applications of this method to Dawn's Mission at Vesta. The low-thrust transfer across the 1 orbit to 1 Vesta rotational period required during the both transfers is unprecedented and inherently unstable. Discovery of transfers with sufficient piecewise open loop stability is critical to the mission successfully reaching and escaping LAMO. The orbital period varies from 12 hours (HAMO) to 4 hours (LAMO) during the transfer. The 1:1 resonance occurs at an orbital period of near 5.34 hours. Other significant, though somewhat weaker resonances, also occur during the transfer. Of particular interest is the 3 orbits to 4 Vesta rotations (3:4) resonance which occurs near an orbital period of 7.12 hours.

Since the period varies from 12 hours (HAMO, HAMO2) to 4 hours (LAMO) during the transfers, a single state cloud template is required to analyze the full HAMO to LAMO and LAMO to HAMO2 transfers. The rule of thumb is that a single state cloud shape (in the RTN frame) developed as described above is adequate for orbital periods within a factor of two of the period around which the covariance was estimated. So a covariance developed at an orbital period of say 7 hours will cover periods varying between 12 hours (HAMO) to 4 hours (LAMO). Despite the crude nature of the state cloud shapes developed at significantly different orbital periods, it appears adequate. It was found that using covariances based on closer orbital periods did not qualitatively change the stability results.

A characteristic covariance was developed for the LAMO to HAMO2 transfer using the method described above. Specifically, a 3 day propagation was was used under uncertainty models approximating the expected mission performance. The resulting covariance is provided in Table 1. The upper left 3 x 3 position submatrix norm of the covariance in Table 1 is 4.1385 [km]. The lower right 3 x 3 velocity submatrix norm of the covariance in Table 1 is 1.4708 [m/s].

Table 1. HAMO to LAMO characteristic covariance in the RTN frame [km,s]

0.08326	-1.15255	-0.01523	0.00041	1.048e-05	1.207e-05
-1.15255	17.0463	0.21288	-0.00606	-0.00016	-0.00019
-0.01523	0.21288	0.00448	-7.61320e-5	-2.08386e-6	-2.90517e-6
0.00041	-0.00606	-7.6132e-5	2.1596e-6	6.0014e-8	6.7631e-8
1.0480e-5	-0.00016	-2.0838e-6	6.0014e-8	2.0984e-9	2.0116e-9
1.2074e-5	-0.00019	-2.9051e-6	6.7631e-8	2.0116e-9	3.3880e-9

The first six cloud states are derived from the covariance in Table 1 as follows

$$\Delta State_i = COV_{cholesky} \cdot EV_i \cdot s \qquad for\ i = 1, 2, 3, 4, 5, 6 \tag{1}$$

where $COV_{cholesky}$ is the lower triangular Cholesky factor of the covariance, EV_i is the i^{th} eigen vector of the covariance, and s is the "sample depth" (or sigma depth). In this analysis, $s = 1$ meaning the state cloud is of size 1 standard deviation according to the covariance used. The remaining 6 cloud states are

$$\Delta State_i = -\Delta State_{i-6} \qquad for\ i = 7, 8, 9, 10, 11, 12. \tag{2}$$

The 12 state cloud derived from this covariance is plotted in Figure 3.

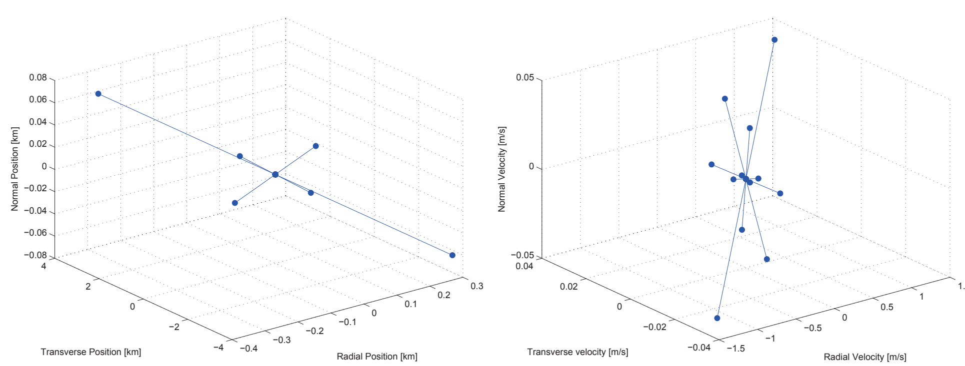

Figure 3. The 12 position and velocity perturbations used in the HAMO to LAMO analysis.

The derived state cloud was used at intervals of 1 day during the entire LAMO to HAMO2 transfer 1 which requires a duration of 33 days. Each cloud was propagated to the end of the transfer. There are a number of ways to present the data resulting from the process described here. An example is a “tube plot” given in Figure 4. Each tube in Figure 4 represents the time evolution of one of the state clouds centered on the reference trajectory 1. Time advances from right to left. The tube horizontal radius represents the maximum spatial divergence of a state cloud. The vertical radius of each tube represents the maximum velocity divergence of the state cloud. Both the spatial and velocity divergences are normalized to be at most 1. The normalization values in this case are 1,874 [km] and 374.4 [m/s] respectively. The state cloud centered at the start of the transfer is represented by the upper right-most tube. This tube is longest because the states that it represents are propagated the longest: from the transfer start to the transfer end. Subsequent tubes are shorter since they begin at intermediate times later in the transfer at 1 day intervals. Relatively stable portions of

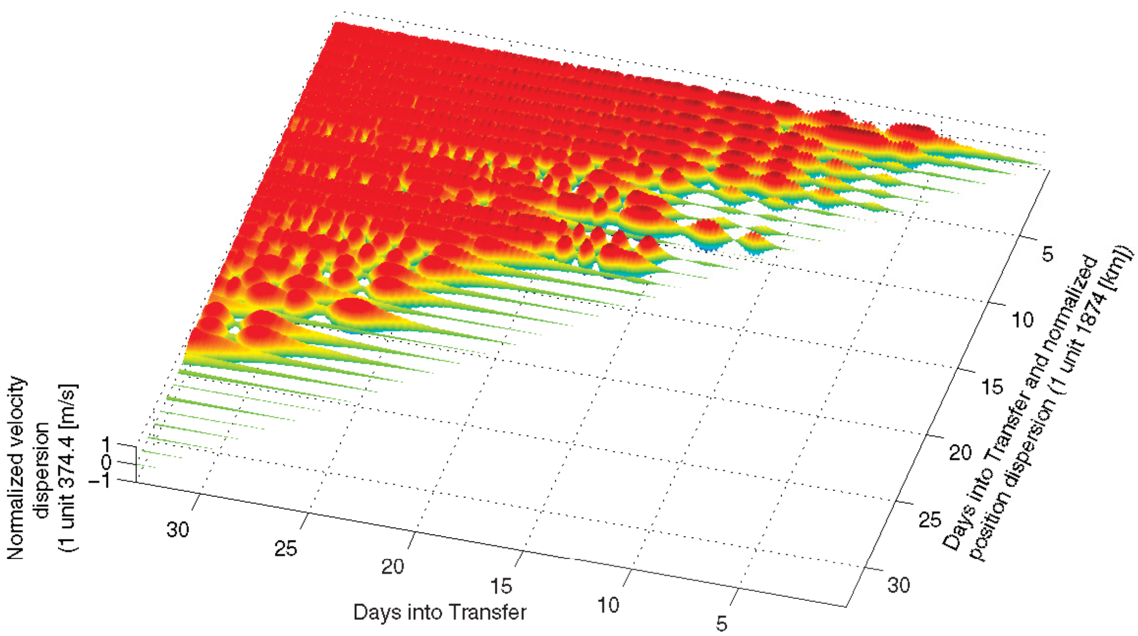

Figure 4. The time evolution of all state clouds during a candidate LAMO to HAMO2 transfer number 1.

the transfer 1 are indicated by long sharp points at the start of a tube. Unstable portions of the transfer are indicated by rapidly increasing radii. For example, the tubes just below and above 10 days into the transfer indicate low powered flight stability. This time is when the transfer passes through the 1:1 resonance. Figure 5 indicates the oscillating orbital period as a function of days into transfer 1. Figure 5 indicates we are near the 1:1 resonance between 5 and 15 days into the transfer. Also passage through the 5:4 and 3:4 resonances at 1 and 20 days into the transfer are less stable based on Figure 4. Plots like the one in Figure 4 can accurately predict the qualitative results of much more complex and computationally expensive analysis involving Monte Carlo simulation of both trajectory optimization and sampling of operational uncertainties as described in Reference 1. However, to begin to predict the results of the much more expensive analysis, several experiments are necessary to see what patterns in the tube plots result in difficulty for the Monte Carlo simulation. For comparison, another transfer (transfer “2”) from LAMO to HAMO2 using different time line,

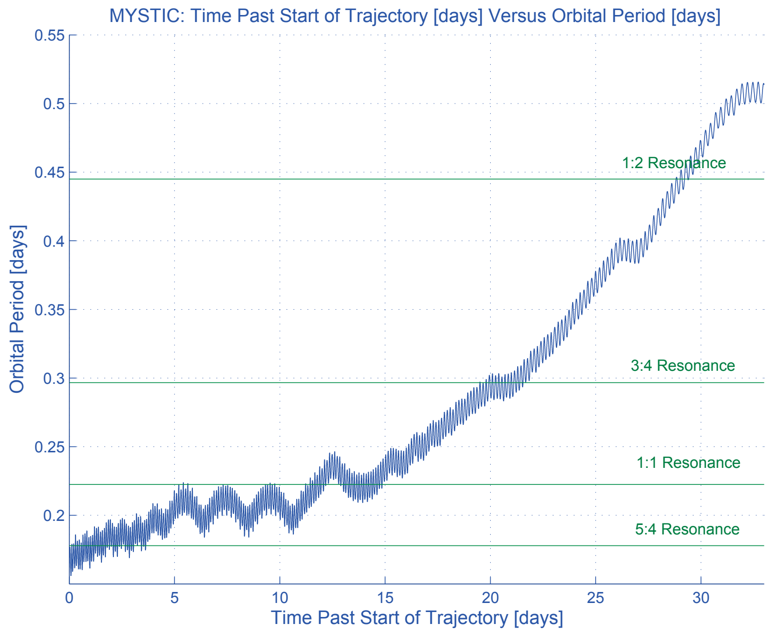

Figure 5. The oscillating orbital period as a function of days into transfer 1. Various orbital - Vesta rotation resonances are indicated.

initial state, and targeting produced a different "tube plot" - see Figure 6. The second transfer (Figure 6) is generally more stable than the first (Figure 4). Particularly when passing through the nearly unavoidable instability of the 1:1 resonance. To facilitate comparison of the stability of transfers 1 and 2, see Figure 7.

Figure 7 compares the time it takes for a state cloud centered at different times around transfer 1 (blue line) and transfer 2 (green line) to grow to a maximum extent of 400 kilometers away from the reference trajectory. For example, at 5 days into transfer 1, a cloud will diverge 400 kilometers from the reference in under 4 days during open loop powered flight. At 5 days into transfer 2, it requires nearly 6 days for a state cloud to diverge more than 400 kilometers from the reference trajectory. Generally, it clear from Figure 7 that transfer 2 is more stable in powered flight than is transfer 1. Most importantly, transfer 2 happens to exhibit significantly more stability near the 1:1 resonance. Most transfers that fail the Monte Carlo optimization test, fail near the 1:1 resonance.

The 1:1 resonance occurs between days 5 and 15 in both transfers. However, even the significantly more stable transfer 2 in Figure 6 exhibits increased divergence near the 1:1 resonance. It has been discovered through experience that successful passage through the 1:1 resonance using the very low control authority of ion propulsion must rely on unstable orbits for part of the transfer. Dawn does not have the thrust magnitude available to push through the resonance without regard to careful phasing with the body rotation. It is easy to apply a simple closed loop control law of thrust with or against the Vesta relative velocity vector and never pass through the resonant altitude. In this situation, the thrust energy is completely transferred to Vesta's rotational energy through the resonant coupling with the gravity field. It remains uncertain as to how to target a reference trajectory so that it will have a high likelihood of being stable enough to fly. A great many considerations (in addition to stability) are necessary when constructing a reference trajectory for Dawn operations at Vesta.[7] Subtle changes in reference transfer targeting result in significant changes in stability.

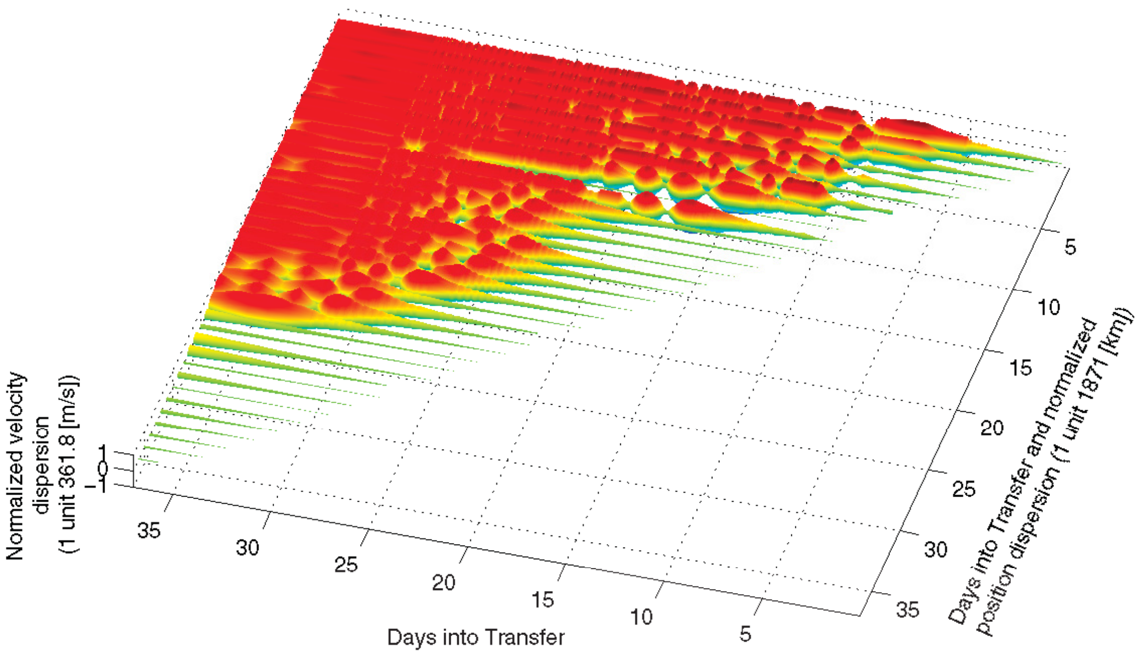

Figure 6. The time evolution of all state clouds during a candidate LAMO to HAMO2 transfer "2".

The current strategy is to generate multiple reference transfers with slight variation and rapidly screen each for stability using the method described here. Once a few candidates with apparently good stability characteristics are found, the computationally expensive Monte Carlo process[2] with optimization and nonlinear propagation is used to verify the robustness of the transfer.

CONCLUSION

The reliability of low-thrust trajectories between science orbits around large asteroids must be evaluated subject to the unavoidable uncertainties of orbit determination, asteroid physical parameters, momentum de-saturation maneuvers, and transfer maneuver execution error. Also rapid evaluation of likely stability is necessary during the severe time limitations present during actual operations. Often many different transfers must be evaluated to find ones that are acceptable for operational constraints and powered flight stability. The brute force method of using nonlinear statistical modeling coupled with design optimization of transfers is very computationally expensive. This paper presents a relatively simple and computationally inexpensive way to measure orbital stability to trajectories undergoing continuously powered low-thrust flight. Trajectories that are stable using this measure have been shown to be stable under the combined uncertainties expected during operations.[2] The measure was applied to Dawn's planned HAMO to LAMO transfer.

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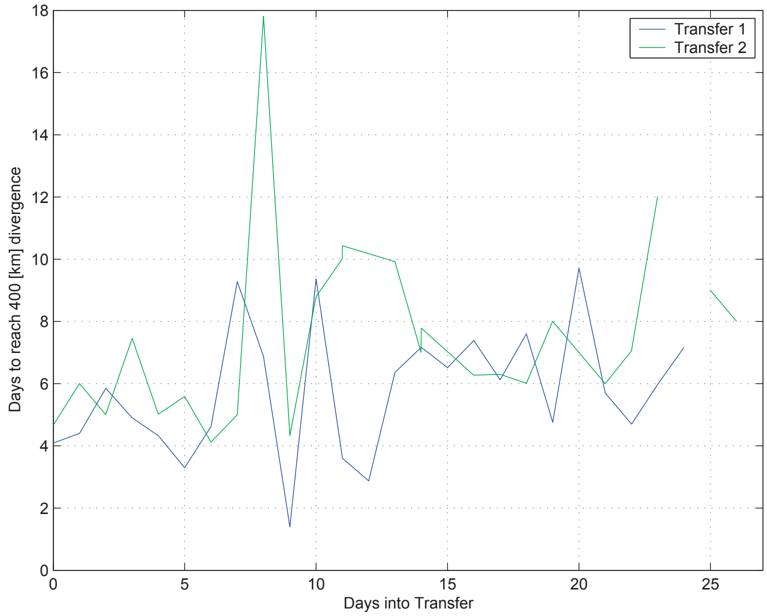

Figure 7. A comparison of transfers 1 and 2. the time it takes state clouds to grow to at least 400 km in extent transfer "2".

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